Dynamic Capital Structure Adjustment and the Impact of Fractional Dependent Variables

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Abstract

Capital structure research using dynamic partial adjustment models aims at estimating the speed of adjustment towards target leverage, as a test of the trade-off theory. Using Monte Carlo simulations, we demonstrate that common estimators usually applied to this end (like OLS, fixed effects, or first-difference GMM estimators) are severely biased since they ignore the fact that debt ratios are fractional, i.e. bounded between 0% and 100%. We propose an estimator designed to be consistent in the context of unbalanced dynamic panel data with a fractional dependent variable (DPF estimator) and examine its statistical properties.

Having established the unbiasedness of the DPF estimator in the context of capital structure, we estimate the speed of adjustment for a typical sample of U.S. firms over the period 1965 to 2008. The resulting estimate of 26% is in the middle of the range of adjustment speeds reported in previous studies. The associated half-life of leverage shocks of about 2.5 years limits—but does not rule out—the economic relevance of the trade-off theory.

Keywords: Capital structure, partial adjustment models, speed of adjustment, fractional dependent variables, Monte Carlo simulation, dynamic panel models.

JEL classification codes: C15, C23, C34, C52, G32.

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The foremost empirical test to differentiate between competing theories regarding the determinants of corporate capital structures is to test for adjustment behavior to leverage shocks. Huang and Ritter (2009) state in this context that “[speed of adjustment is] perhaps the most important issue in capital structure today.” Correspondingly, a large strand of the empirical literature focuses on estimating the speed of adjustment, where very low values would contradict the relevance of the trade-off theory and favor alternative explanations which do not predict adjustment behavior to shocks, like the pecking order theory or market timing.\(^1\)

Estimating the corresponding dynamic partial adjustment models is econometrically a challenging task, however, because typically

(i) financial data on companies are panel data (i.e. few time-series observations but a large number of firm observations),

(ii) adjustments are not instantaneous, which requires a lagged dependent variable in the regressions,

(iii) the data are unbalanced due to IPOs, delistings, bankruptcies, etc., and

(iv) the variable of interest, the (market) debt ratio, is fractional (i.e. limited between 0 and 1).

In this study, we use Monte Carlo simulations to show that commonly applied estimators for adjustment speed estimation (like pooled OLS, Fama and MacBeth (1973) regressions, fixed effects, or first-difference GMM estimators) are severely biased since they do not account for the fractional nature of debt ratios. As our main contribution, we then propose

\(^1\)Note that a finding of zero adjustment speed cannot definitively rule out theoretic trade-off models, since for some model parameterizations (e.g. regarding adjustment costs), these models might predict zero adjustment, although the trade-off rationale is the built-in driving feature of firm behavior. As an example, see Strebulaev (2007). Welch (2004) notes, however, that such a frictions-based explanation of no capital structure adjustment is hard to reconcile with the fact that the volatility of leverage changes per year is fairly high, at about 25% p.a.
a new estimator, which is unbiased and asymptotically consistent in the context of unbalanced dynamic panel data with a fractional dependent variable (DPF estimator). Further Monte Carlo simulations are used to examine the statistical properties of the DPF estimator, complemented by its application to the Compustat sample of U.S. companies typically used in previous studies, covering the period from 1965 to 2008.

Fractional variables, i.e. variables bounded between zero and one, can be found in many economic contexts, e.g. market shares or debt ratios. Most standard econometric models are inappropriate for estimation if the dependent variable is bounded to some interval including the endpoints, e.g. [0,1], and where limit values are not probabilistic outcomes. It is obvious that such a limited variable cannot be normally or symmetrically distributed as common estimators assume, and these estimators tend to attribute the fact that observed debt ratios remain in the [0,1] interval erroneously to be due to mean reversion (Iliev and Welch (2010)). The DPF estimator proposed in this paper is based on a doubly-censored Tobit-estimator suggested by Loudermilk (2007), which takes the fractional and endogenous lagged dependent variable, and the incidence of unobserved firm heterogeneity (i.e. the fixed effects) into account by using a latent variable approach and assuming a parametric specification for the fixed effects distribution.\(^2\) However, the Loudermilk (2007) estimator and related alternatives—e.g. that suggested by Papke and Wooldridge (2008)—suffer from the fact that these estimators require balanced panel data for estimation. Data on corporate finance basically never meet this requirement, due to frequent IPOs, M&As, bankruptcies, etc. This renders these estimators inapplicable to capital structure analysis.\(^3\)

\(^2\)The notion of censored debt ratios can be intuitively best explained with a latent variable reflecting firms’ debt capacities, see Section II.A.

\(^3\)Note that “balancing” the corporate data by including only those firms into the sample which have a consistently long observation period is infeasible since it would introduce severe selection biases. Entrance and exit to the sample due to bankruptcies, M&As, etc., are events systematically related to many of the
To overcome this issue, we extend the Loudermilk (2007) estimator to unbalanced panel data by adjusting the assumed fixed effects distribution accordingly. Our Monte Carlo simulations show for realistic parameters in the capital structure context (the parameters are calibrated to the Compustat data) that the DPF data-generating process generates realistic firm leverage distribution characteristics, and that the estimator is unbiased for these data. Standard estimators, however, show positive (fixed effects and Arellano and Bond (1991) dynamic panel estimator) or negative biases (OLS, Fama/MacBeth regression) of up to 50% and 90% of the true value.

We also check for the robustness of the DPF estimation results in the capital structure context, addressing the estimator’s requirement to explicitly specify a distributional assumption for the fixed effects. We analyze mis-specification of target leverage by including irrelevant regressors in the specification. The DPF estimator is robust to this common type of mis-specification in applied work. In another robustness check, we test the estimator’s characteristics under a specification test suggested by Iliev and Welch (2010). They suggest testing estimators in the capital structure context using resampled firm data from Compustat, thereby maintaining the empirical distribution of observed leverage. The DPF estimator performs well in this setup, even if target leverage is subject to measurement error or partly unknown.

We then apply the DPF estimator to the Compustat firm universe over the periods 1965 to 2008 and 1965 to 2001, basically replicating the study by Flannery and Rangan (2006) to allow for direct comparisons. For this data, standard estimators (including the fixed effects instrumental variables estimator advocated by Flannery and Rangan (2006)) yield estimates for the speed of adjustment in the range of 15% to 40%, similar to the heterogene-

important capital structure determinants like profitability, a company’s rating and so on, and maybe to the capital structure itself.
ity of estimates reported across other studies (e.g. Welch (2004)). Using the DPF estimator, we find a speed of adjustment estimate in the middle of this range of about 26%, which corresponds to a half-life of leverage shocks of about 2.5 years. As noted by Iliev and Welch (2010), this speed of adjustment is economically in line with “active managerial interventions”. Thus, our evidence does not rule out the economic relevance of the trade-off theory, but illustrates that alternative explanations for capital structure choices, like the pecking order theory or market-timing, are required to explain most of the observed variability in firm capital structures.

The remainder of the paper is structured as follows. Section I provides an overview of current empirical evidence on tests of dynamic trade-off theories. Furthermore, methodological issues regarding the econometrics of partial adjustment models will be discussed, focusing on the impact of fractional dependent variables. Section II contains Monte Carlo evidence examining the potential bias of non-fractional estimation methods, and providing an analysis of the statistical properties of the DPF estimator. Section III contains the robustness checks with respect to mis-specification of the regressors and testing for non-adjustment using resampled data. In Section IV, we apply the estimator to the typical corporate finance data used in previous studies. In particular, the partial adjustment model studied by Flannery and Rangan (2006) is estimated with fractional and non-fractional methods for their Compustat sample of U.S. firms. Section V concludes.

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4 An AR(1) process has an exponentially declining response function to shocks. Half-life is the time the process needs to close the gap between the actual debt ratio and the target by 50%, after a one unit shock to the error term. Thus, half-life is calculated as \(\log(0.5)/\log(1 – \text{Speed of adjustment})\); see Iliev and Welch (2010).
I. Dynamic Trade-Off and the Speed of Adjustment

A. Evidence

In the dynamic version of the classic trade-off theory, target leverage can be time-varying. If there are (for any reason) deviations from the optimal capital structure, the theory\(^5\) states that there will be adjustment towards the “optimal” target. Capital structure research using dynamic partial adjustment models then strives to estimate the speed of adjustment.

In what follows, we use the recent study by Flannery and Rangan (2006) as a benchmark for comparisons. They analyze whether U.S. firms have long-run target capital structures and if so, how fast they adjust to this target. In comparison to prior studies, they put special emphases on the econometric methods and the model specification, underscoring the need to take the panel nature of the data into account.

In Flannery and Rangan (2006), target leverage of firm \(i\) at time \(t+1\) is determined by a vector of firm characteristics \(X_{it}\) that are related to the trade-off between the costs and benefits of debt and equity in different capital structures. Target leverage is given by

\[
MDR^*_{i,t+1} = \gamma X_{it},
\]

where \(\gamma\) is a coefficient vector, and \(MDR\) denotes market debt ratio. For firms to have a target capital structure, there must be at least some elements of \(\gamma\) different from zero.

The partial adjustment model of Flannery and Rangan (2006) has the form

\[
MDR_{i,t+1} - MDR_{it} = \lambda \left( MDR^*_{i,t+1} - MDR_{it} \right) + \epsilon_{i,t+1}.
\]

\(^5\)For more on the theory of dynamic trade-off, see the surveys by Frank and Goyal (2008) or Titman and Tsyplakov (2007).
Plugging the target leverage function (1) into (2), the rearranged partial adjustment model is

$$ MDR_{i,t+1} = (\lambda \gamma) X_{it} + (1 - \lambda) MDR_{it} + c_i + \epsilon_{i,t+1}, $$

where $\lambda$ is the adjustment speed coefficient, $c_i$ a time-invariant unobserved variable (firm fixed effect), and $\epsilon_{i,t+1}$ an error term. The speed of adjustment is assumed to be the same for all firms and captures the extent to which deviations from optimal leverage are eliminated in each period: if the current deviation from the target debt ratio marginally increases, the difference between the future and the current debt ratio increases by $\lambda$. If $\lambda = 0$, the speed of adjustment is zero, that is, there is no adjustment towards target leverage at all. If $\lambda = 1$, the speed of adjustment is infinitely high, that is, the debt ratio is always at its target value.

According to the estimated coefficient on $MDR_{it}$, the speed of adjustment is $\lambda = 1 - 0.656 = 0.344$ in the Flannery and Rangan (2006) analysis for U.S. firms, significantly different from zero in a statistical and economic sense. Hence, about 34% of the deviation from optimal leverage is eliminated in each period, taking about three years (or a half-life of 1.7 years) for the average firm to adjust to its target capital structure following shocks.

More generally, empirical estimates of the speed of adjustment are within a rather large range from 0% up to about 40%. For example, Fama and French (2002) or Kayhan and Titman (2007) find very low adjustment speeds of 7 to 18%, Lemmon, Roberts, and Zender (2008) and Huang and Ritter (2009) estimate about 25%, while Leary and Roberts (2005) and Alti (2006) find relatively fast adjustment similar to Flannery and Rangan (2006). The study by Welch (2004) uses a different methodology than the other studies by testing adjust-
ment behavior to leverage shocks from stock price changes. He basically finds no adjustment at all.\footnote{Some other studies like Titman and Tsyplakov (2007) or Strebulaev (2007) combine a theoretical and empirical analysis. The theoretical models are constructed such that they reflect a dynamic trade-off between equity and debt. Firm behavior is then simulated under “realistically” parameterized models, generating data with known properties of the firms’ capital structure determinants. In these simulated universes, the speed of adjustment is fairly low, with a speed of adjustment of about 7%.}

All these empirical analyses have in common, however, that unlike this study, they ignore the fractional nature of debt ratios as the variable of interest. As we will show below, this leads to severely biased estimates of the speed of adjustment.

(i) financial data on companies are panel data (i.e. few time-series observations but a large number of firm observations),

(ii) adjustments are not instantaneous, which requires a lagged dependent variable in the regressions,

(iii) the data are unbalanced due to IPOs, delistings, bankruptcies, etc., and

(iv) the variable of interest, the (market) debt ratio, is fractional (i.e. limited between 0 and 1).

B. Methodological Issues

Estimating dynamic partial adjustment models is econometrically a challenging task because typically financial data on companies are (i) unbalanced panel data, (ii) the model takes adjustment over time into account (i.e. a lagged dependent variable is included as a regressor), and (iii) the dependent variable is fractional. One possible and likely reason for the mixed evidence on the speed of adjustment in previous studies is that these studies rely on different estimators, which are biased due to one or several of these econometric issues. For example, the fixed effects estimator (FE) takes the panel nature of the data
into account, but the existence of lagged dependent variables leads to a correlation between the error term and the explanatory variables (i.e. endogeneity), which renders FE biased for fixed $T$ (and OLS or Fama/MacBeth regressions as well). The instrumental-variables fixed effects estimator used by Flannery and Rangan (2006) or the first difference GMM estimator by Arellano/Bond are consistent with a lagged dependent variable in the panel context. Still, even these estimators will be biased, since they do not take the fractional nature of the dependent variable into account.

As also noted by Iliev and Welch (2010), assuming an unconstrained (normally distributed) leverage process is conceptually wrong because debt ratios are economically restricted to the interval $[0,1]$. Moreover, an unconstrained process is not even a good empirical approximation for observed leverage patterns, since its variance is way too high to not generate an unrealistically huge amount of observations at the borders of the feasible debt ratio range.

Most standard econometric models are inappropriate for estimation if observed debt ratios are limited and can take the limiting values zero and one. Methods that account for the special nature of fractional dependent variables are scarce because in the nonlinear panel context, except for special cases, it is not possible to separate the unobserved fixed effects from the maximum-likelihood estimates of the explanatory variables’ coefficients (the so-called incidental parameters problem). Since no known transformation will eliminate the unobserved heterogeneity, a semi-parametric approach needs to be used for estimation, or the distribution of the unobserved heterogeneity (the fixed effects) needs to be specified explicitly (see Baltagi (2005), or Loudermilk (2007)).

The latter approach is taken by Papke and Wooldridge (2008) for small $T$, large $N$ panel data, allowing for unobserved heterogeneity and endogenous regressors. However, the case
of a lagged dependent variable is not considered. Loudermilk (2007) develops a Tobit specification for fractional response variables allowing for corner solution observations at both 0 and 1 ("doubly-censored Tobit") with lagged dependent variable and unobserved heterogeneity. The drawback of the Loudermilk (2007) estimator is that it requires balanced data, which as mentioned before renders it inapplicable to corporate finance data, where entry and exit to the data is very frequent and related to either the firm’s capital structure itself, or its determinants. The DPF estimator suggested in this study builds on Loudermilk (2007), but changes the specification of the presumed fixed effects distribution such that it allows for unbalanced panel data with a lagged dependent variable—the typical data structure encountered in capital structure research.

II. Monte Carlo Evidence: Fractional Dependent Variables in Dynamic Panel Models

A. DPF Estimator and Design of the Monte Carlo Simulations

In this section, Monte Carlo simulations are conducted in order to analyze the unbiasedness of different estimation methods typically employed in empirical analyses of the speed of adjustment of capital structures. The Monte Carlo simulations are based on our extension of the fractional response model by Loudermilk (2007), which we call the dynamic panel fractional estimator (DPF). We will use the data-generating process underlying this estimator for the Monte Carlo design, which allows us to show that (i) the resulting characteristics of the data closely resemble the observed data on firm capital structures,\(^7\) (ii) the DPF estima-

\(^7\)We orient our parameter choices for the data-generating process to the Compustat sample of U.S. firms from 1965 to 2008, also see Section IV.
tor is consistent in this setting, and (iii) all estimators commonly used to estimate dynamic partial adjustment models in the capital structure literature are severely biased.

The DPF estimator\(^8\) is a doubly-censored Tobit specification allowing for corner solution observations at both zero and one with a lagged dependent variable and unobserved heterogeneity. It is estimated by maximum likelihood.

To take the fractional nature of the dependent variable into account, the DPF estimator employs a latent variable specification. The latent (unobservable) variable in the small \(T\), large \(N\) panel model is given by

\[
y^*_t = z_t \gamma + y_{i,t-1} (1 - \lambda) + c_i + u_{it},
\]

where \(z_{it}\) is a set of exogenous regressors, \(u_{it} \sim N(0, \sigma_u^2)\) an error term, and the observable doubly-censored dependent variable with two possible corner outcomes is

\[
y_t = \begin{cases} 
0 & \text{if } y^*_t \leq 0 \\
y^*_t & \text{if } 0 < y^*_t < 1 \\
1 & \text{if } y^*_t \geq 1.
\end{cases}
\]

As explained above, the model requires the specification of a conditional distribution for the unobserved heterogeneity \(c_i\) (i.e. the fixed-effects), and variables other than the lagged dependent variable must be strictly exogenous. The time-invariant unobserved variable is

\[
c_i = a_0 + a_1 y_{i0} + E(z_{i,})a_2 + a_i,
\]

\(^8\)The DPF estimator is easily implemented in Stata, see Appendix A for details.
with error term $a_i \sim N(0, \sigma_a^2)$ and $E(z_{it})$ being the expected value of $z_{it}$ over $t$. This choice of the distribution for the fixed effect $c_i$ allows a correlation structure between the regressors of the model and the fixed effect.\footnote{Without this correlation, the fixed effect would actually be a random effect.} The term $\alpha_1 y_{i0}$ deals with the initial conditions problem in dynamic nonlinear panel data as suggested by Wooldridge (2005). Note that we have adjusted the distributional assumption for the fixed effects compared to Loudermilk (2007), who incorporates all pooled observations of the exogenous explanatory variables into the fixed effects specification. This allows in principle for a richer specification, but it increases the number of parameters to be estimated significantly, and it is because of this structure that the estimation requires balanced data. Our modification, rather, assumes that the fixed effects distribution depends on the expected value, $E(\cdot)$, of the exogenous variables, which appears as a reasonable simplification (and a common choice, see e.g. Chamberlain (1980)) which, however, is robust to (randomly) missing values in the $z$-matrix.

In economic terms, a latent variable reflecting a firm’s debt ratio can be interpreted as the firm’s debt capacity. The debt capacity can be above 100% of current total assets, for example if the firm is so profitable that expected distress costs are negligible and the tax shield effect of interest payments on debt would only be fully exploited with a much larger debt level. Similarly, a negative debt capacity can arise if a firm is subject to severe agency costs and opacity, like for example start-up companies investing in R&D on new technologies.

In the capital structure context, $y_{it}$ is the observable debt ratio bounded between zero and one and $\lambda$ is equivalent to the speed of adjustment in partial adjustment model (3). The corresponding firm characteristics are given by a matrix $z_{it}$. The data in this simulation can be generated by an economic model based on a dynamic trade-off theory. However, the
simulation design and data generation is held very general in order to focus on the methodological issues associated with neglecting the fractional nature of dependent variables in this setting.

The parameter choices in the simulations are as follows. The single (exogenous) fixed regressor is \( z_{it} \sim U(-1,1) \) with \( \gamma = 0.5 \). The initial latent variable is given by

\[
y_{i0}^* = z_{i0}\gamma + \alpha_0 + \frac{1}{T} \sum_{t=1}^{T} z_{it}\alpha_2 + a_i + u_{i0},
\]

with \( u_{it} \sim N(0,\sigma_u^2) \), where \( \sigma_u = 0.1 \). The time-invariant unobserved variable is

\[
c_i = \alpha_0 + \alpha_1 y_{i0} + \frac{1}{T} \sum_{t=1}^{T} z_{it}\alpha_2 + a_i,
\]

with \( a_i \sim N(0,\sigma_a^2) \), where \( \sigma_a = 0.1 \), \( \alpha_0 = 0.1 \), \( \alpha_1 = 0.1 \), and \( \alpha_2 = -0.25 \).

**B. Data Characteristics**

It will not come as a surprise if the DPF estimator performs well under our specification of the simulated data, simply because it is designed to do so in this situation. However, we will in this section demonstrate that the simulated data is comparable to the real data on debt ratios used in Section IV, such that we can learn about the biases of other commonly applied estimators in the literature. We will consider the consequences of a mis-specification of the data-generating process for the statistical properties of the DPF estimator in Section III.

When comparing simulated data to real data on leverage, it is important to stress that we do not know the true distribution underlying real debt ratios. In fact, best fitting our data-generating process to the real data corresponds to performing the estimation reported
in Section IV, where we apply the DPF estimator. However, for the design of the Monte Carlo study, we do not know the best-fitting parameters a priori, and actually we want to compare the characteristics of different estimators over a wide range of values for the speed of adjustment parameter $\lambda$. This parameter in turn essentially determines how many corner observations will result, which in turn has a major impact on the mean of the leverage distribution. One should expect to find that the shape of the distribution resembles the shape of the true distribution, however. In particular, this implies that we must compare the simulated and true data with regard to the overall shape, the standard deviation of the debt ratios, and the number of zero observations, because the latter represents the major point of censoring in the real data.

**Figure 1: Histograms for the Frequency Distributions of One Monte Carlo Data Set and Real MDR Based on Compustat**

On the left hand side, the figure shows the relative frequency distribution of the real data market debt ratios in the Compustat sample for the period 1965 to 2008. On the right hand side, the relative frequency distribution of simulated market debt ratios with speed of adjustment $\lambda = 0.3$, that is 30%.

Figure 1 shows a histogram of the frequency distribution of market debt ratios for the Compustat firm universe (left hand side) and for one random data set used for the Monte
Carlo simulations, assuming a speed of adjustment of $\lambda = 0.3$, that is 30%. As can be seen, the simulated data mimics the real data in terms of the large peak at the lower censoring point of zero (i.e. no indebtedness), and the decay of leverage towards larger values. The distribution differs regarding the upper censoring limit of one, where the simulated data has significantly more extreme observations at the border point (6.5% versus 0, see Table I). This depends strongly, however, on the choice of the adjustment speed, which we do not know for the true data, of course.

Table I: Average Sample Statistics in a Small Sample Setup

This table shows average sample statistics for the Monte Carlo simulations of the DPF model as parameterized in Section II.A. “Mean” stands for the mean value of all observations of the variable for the respective choice of the true speed of adjustment $\lambda$. The standard deviation of the variable is in parentheses. The next to last row gives the percentage of all pooled observations of the latent variable lower than zero and then set to zero. The last row gives the percentage of all pooled observations of the latent variable larger than one and then set to one.

<table>
<thead>
<tr>
<th>True $\lambda$</th>
<th>0.0</th>
<th>0.2</th>
<th>0.3</th>
<th>0.5</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{it}$ Mean</td>
<td>0.509</td>
<td>0.389</td>
<td>0.329</td>
<td>0.244</td>
<td>0.175</td>
<td>0.149</td>
</tr>
<tr>
<td></td>
<td>(0.245)</td>
<td>(0.194)</td>
<td>(0.173)</td>
<td>(0.148)</td>
<td>(0.133)</td>
<td>(0.128)</td>
</tr>
<tr>
<td>$z_{it}$ Mean</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.284)</td>
<td>(0.284)</td>
<td>(0.284)</td>
<td>(0.284)</td>
<td>(0.284)</td>
<td>(0.284)</td>
</tr>
<tr>
<td>$c_i$ Mean</td>
<td>0.114</td>
<td>0.114</td>
<td>0.114</td>
<td>0.114</td>
<td>0.114</td>
<td>0.114</td>
</tr>
<tr>
<td></td>
<td>(0.111)</td>
<td>(0.110)</td>
<td>(0.110)</td>
<td>(0.111)</td>
<td>(0.111)</td>
<td>(0.110)</td>
</tr>
<tr>
<td>$y_{it} = 0$ Obs. in %</td>
<td>0.128</td>
<td>0.151</td>
<td>0.165</td>
<td>0.200</td>
<td>0.258</td>
<td>0.297</td>
</tr>
<tr>
<td>$y_{it} = 1$ Obs. in %</td>
<td>0.220</td>
<td>0.063</td>
<td>0.025</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table I shows sample statistics for the simulated data under different values for the speed of adjustment parameter $\lambda$. Comparing this with the descriptive statistics provided in Section IV.A further illustrates the similarity between the simulated and the observed data, but also shows that the simulated data is no perfect fit. For example, the standard deviation of the simulated debt ratios is 17% versus an empirical estimate of 25%, and the
Table II: Standard Error Adjustments in Finance Studies

Estimators and adjustment of standard errors in 207 studies relying on panel data and being published in top finance journals, as reported in Petersen (2009).

<table>
<thead>
<tr>
<th>Estimation Method and Standard Error Correction</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Adjustment</td>
<td>42</td>
</tr>
<tr>
<td>Adjustment</td>
<td></td>
</tr>
<tr>
<td>- Fama/MacBeth</td>
<td>34</td>
</tr>
<tr>
<td>- Fixed Effects</td>
<td>29</td>
</tr>
<tr>
<td>- OLS and White</td>
<td>23</td>
</tr>
<tr>
<td>- OLS and Newey-West</td>
<td>7</td>
</tr>
</tbody>
</table>

The number of zero-observations is 16.5% versus 11%. The average correlation between the exogenous regressor and the unobserved heterogeneity $\text{Corr}(z_{it}, c_i)$ is about 0.33.

Overall, these patterns suggest that the general data-generating process assumed for the DPF estimator is able to generate leverage distributions similar to the real world distribution.

C. Estimation Methods

We base our choice of estimators to be analyzed regarding their unbiasedness on the results by Petersen (2009), who gathered information about estimation of standard errors in a panel context from 207 finance papers published in major finance journals (i.e. J. Finance, J. Finan. Econ., Rev. Finan. Stud., within the period 2001 to 2004). Table II summarizes his results on the employed estimation methods. Many studies of partial adjustment models estimating the speed of adjustment have used these estimators or similar variants thereof.

A further method to be considered is the Arellano and Bond (1991) first-difference GMM estimator which is designed for (and thus consistent) with the case of dynamic panel data (but without allowing a fractional dependent variable).
D. Results

Figure 2 provides simulation results for the methods discussed in Subsection II.C. The figure shows the average estimates of the speed of adjustment $\lambda$ for these estimators, which are applied to the simulated data in each run of the Monte Carlo simulations. The horizontal axis shows the true parameter underlying the simulation, the vertical axis the corresponding average of estimated coefficients. The number of Monte Carlo runs is $S = 1000$, the number of cross-sections is $N = 100$, and the number of periods is $T = 8$.

In this small-sample setup, pooled OLS (“OLS”) and the Fama/MacBeth estimator (“Fama-Mac”) severely underestimate the speed of adjustment. For example, with a true $\lambda$ of 0.4, the estimated OLS coefficient is only about 0.2, a 50% bias. The curve representing the estimated coefficients for the Fama/MacBeth estimator is similar to the OLS case, which was to be expected because the Fama/MacBeth estimator is pooled OLS with standard errors corrected for time heterogeneity (instead of firm heterogeneity, as in the data). The fixed effects estimator (“FixEff”) and the Arellano/Bond estimator (“AreBo”) severely overestimate $\lambda$, where this bias gets worse for lower true speeds of adjustment $\lambda$. Finally, the straight (blue) line in between the other parameters represents the average estimate of the DPF estimator. This number coincides almost perfectly with the true value (with a deviation of less than 0.6%, see Table III), demonstrating that the DPF estimator is consistent (even for such a rather small sample of only 100 firms).

The upper panel of Table III shows the deviation of the average estimate for each estimator relative to the true value. In contrast to the figure, the table shows these values only over the range of $\lambda$ from 0.1 to 0.4, in order to focus on the speed of adjustment values which are reasonable given the range of estimates reported in previous studies. In the cases of
Figure 2: Simulation Results with Fractional Dependent Variable, “Small Sample”

The figure shows the average estimates of the speed of adjustment $\lambda$, which corresponds to one minus the coefficient on the lagged dependent variable, i.e. $1 - \lambda$. The latent variable is given by

$$y^*_t = z_{it} \gamma + y_{i,t-1}(1 - \lambda) + c_i + u_{it},$$

with error term $u_{it} \sim N(0, \sigma_u^2)$ and the observable variable with two possible corner outcomes doubly-censored (“adjustment”)

$$y_{it} = \begin{cases} 
0 & \text{if } y^*_t \leq 0 \\
 y^*_t & \text{if } 0 < y^*_t < 1 \\
1 & \text{if } y^*_t \geq 1.
\end{cases}$$

The time-invariant unobserved variable is

$$c_i = \alpha_0 + \alpha_1 y_{i0} + \frac{1}{T} \sum_{t=1}^{T} z_{it} \alpha_2 + a_i,$$

with error term $a_i \sim N(0, \sigma_a^2)$. The model parameter choices are described in Section II.A. The horizontal axis shows the true parameter choice in the simulation and the vertical axis the corresponding estimated coefficient. Results are reported for different estimators: pooled OLS (“OLS”), Fama/MacBeth estimator (“FamaMac”), fixed effects (“FixEff”), Arellano/Bond (“AreBo”), and the DPF estimator suggested in Section II.A (“DPF”). The number of Monte Carlo runs is $S = 1000$, the number of cross-sections is $N = 100$, and the number of periods is $T = 8$. 

Figure 3: Simulation Results with Fractional Dependent Variable, “Large Sample”

The figure shows the average estimates of the speed of adjustment $\lambda$, which corresponds to one minus the coefficient on the lagged dependent variable, i.e. $1 - \lambda$. The latent variable is given by

$$y_{it}^* = z_{it} \gamma + y_{i,t-1}(1 - \lambda) + c_i + u_{it},$$

with error term $u_{it} \sim N(0, \sigma_u^2)$ and the observable variable with two possible corner outcomes doubly-censored (“adjustment”)

$$y_{it} = \begin{cases} 
0 & \text{if } y_{it}^* \leq 0 \\
y_{it}^* & \text{if } 0 < y_{it}^* < 1 \\
1 & \text{if } y_{it}^* \geq 1.
\end{cases}$$

The time-invariant unobserved variable is

$$c_i = \alpha_0 + \alpha_1 y_{i0} + \frac{1}{T} \sum_{t=1}^{T} z_{it} \alpha_2 + a_i,$$

with error term $a_i \sim N(0, \sigma_a^2)$. The model parameter choices are described in Section II.A. The horizontal axis shows the true parameter choice in the simulation and the vertical axis the corresponding estimated coefficient. Results are reported for different estimators: pooled OLS (“OLS”), Fama/MacBeth estimator (“Fama-Mac”), fixed effects (“FixEff”), Arellano/Bond (“ArelBo”), and the DPF estimator suggested in Section II.A (“DPF”). The number of Monte Carlo runs is $S = 1000$, the number of cross-sections is $N = 1000$, and the number of periods is $T = 8$. 
Table III: Average Bias of Estimators

This table shows the deviation of the average estimate for each estimator relative to the true value of $\lambda$ in %. The upper panel contains the small sample simulation biases and the lower panel the large sample simulation biases.

<table>
<thead>
<tr>
<th>True $\lambda$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N = 100$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DPF</td>
<td>0.93</td>
<td>0.61</td>
<td>0.58</td>
<td>0.35</td>
</tr>
<tr>
<td>OLS</td>
<td>-21.84</td>
<td>-49.33</td>
<td>-51.49</td>
<td>-46.89</td>
</tr>
<tr>
<td>Fama/MacBeth</td>
<td>-87.94</td>
<td>-73.10</td>
<td>-62.73</td>
<td>-52.79</td>
</tr>
<tr>
<td>Arellano/Bond</td>
<td>174.27</td>
<td>56.26</td>
<td>28.89</td>
<td>20.38</td>
</tr>
<tr>
<td>Fixed effects</td>
<td>192.86</td>
<td>75.12</td>
<td>44.35</td>
<td>32.17</td>
</tr>
<tr>
<td>Large sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N = 1000$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DPF</td>
<td>-0.59</td>
<td>-0.33</td>
<td>-0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>OLS</td>
<td>-22.87</td>
<td>-50.29</td>
<td>-52.19</td>
<td>-47.50</td>
</tr>
<tr>
<td>Fama/MacBeth</td>
<td>-86.74</td>
<td>-73.41</td>
<td>-62.80</td>
<td>-52.82</td>
</tr>
<tr>
<td>Arellano/Bond</td>
<td>159.87</td>
<td>49.53</td>
<td>23.73</td>
<td>16.06</td>
</tr>
<tr>
<td>Fixed effects</td>
<td>191.60</td>
<td>74.72</td>
<td>43.86</td>
<td>31.90</td>
</tr>
</tbody>
</table>

OLS and Fama/MacBeth the bias goes up to 50% and 90% of underestimation of true value, while fixed effects and the Arellano/Bond estimator reach overestimations of up to 90%.

Figure 3 shows the same simulation exercise in a large sample setting, where we increase the number of firms, $N$, to 1000, but keep the number of time observations (and all other parameters) fixed. This shows the performance of the estimators when the asymptotics get stronger at least for the fixed effects and the Arellano/Bond estimator, since for these, the probability limit is taken over the individuals. There is no reason to expect an improvement of pooled OLS or the Fama/MacBeth estimators because these do not take the panel nature into account at all.

It becomes obvious from Figure 3, that the biasedness result remains unchanged not only for OLS and Fama/MacBeth regressions but also for fixed effects and the Arellano/Bond estimator. As before, the DPF estimator perfectly fits the true value on average and is consistent for large $N$ as well. Hence, the bias of the other estimators is not driven by
the structure of the data in terms of its size, but by the fractional nature of the dependent variable.

Another simulation exercise documented in Figure 4 illustrates this impact of the fractional nature of the dependent variable. To generate this figure we have repeated the analysis of the Monte Carlo simulations, but this time we have applied the estimators to the latent variable, \( y_{it}^* \). Thus, the dependent variable is not fractional and the figure illustrates the bias resulting from analyzing a standard dynamic panel data set with a lagged dependent variable.

Allowing the dependent variable to take on values lower than zero and larger than one changes the estimation results significantly. Pooled OLS and Fama/MacBeth are still highly biased. The fixed effects estimator and the Arellano/Bond estimator are much closer to the true values now. The fixed effects estimator still exhibits a systematic bias over the full range of true \( \lambda \), but the bias is significantly decreased. The Arellano/Bond estimator is even closer to the true values, as to be expected because it is designed for this type of data. The remaining small deviance from the true values is akin to the small sample setting in the simulations and would disappear if we increased the number of cross-sections. The best performing estimator is, however, again the DPF estimator, although it is still configured for censoring at zero or one, which demonstrates some robustness of this estimator against errors in the specification of the censoring limits.

III. Robustness of the DPF Estimator

A. Mis-Specification of Target Leverage

We have already demonstrated that the DPF estimator remains unbiased even if the data is actually not censored (although this outcome might be to some extent driven by
Figure 4: Simulation Results with Unrestricted Dependent Variable, “Small Sample”

The figure shows the average estimates of the speed of adjustment $\lambda$, which corresponds to one minus the coefficient on the lagged dependent variable, i.e. $1 - \lambda$. The latent variable is given by

$$y_{it}^* = z_{it}Y + y_{it-1}(1 - \lambda) + c_i + u_{it},$$

with error term $u_{it} \sim N(0, \sigma_u^2)$ and assumed to be observable to test for estimators’ biases not driven by the fractional nature of the dependent variable. The time-invariant unobserved variable (fixed effect) is

$$c_i = \alpha_0 + \alpha_1 y_{i0} + \frac{1}{T} \sum_{t=1}^{T} z_{it}\alpha_2 + a_i,$$

with error term $a_i \sim N(0, \sigma_a^2)$. The model parameter choices are described in Section II.A. The horizontal axis shows the true parameter choice in the simulation and the vertical axis the corresponding estimated coefficient. Results are reported for different estimators: pooled OLS (“OLS”), Fama/MacBeth regressions (“FamaMac”), fixed effects (“FixEff”), Arellano/Bond (“ArelBo”), and the DPF estimator suggested in Section II.A (“DPF”). The number of Monte Carlo runs is $S = 1000$, the number of cross-sections is $N = 100$, and the number of periods is $T = 8$. 
the specifics of the simulation parameters). However, the one potential disadvantage of the DPF estimator is that it requires a parametric specification of the fixed effects distribution. Since this distribution is not known in reality (hence the name “unobserved heterogeneity”), it is important to test the robustness of the estimator against a mis-specification of this function. We cannot examine the robustness of the DPF estimator in all possible cases though, because there are too many scenarios to be considered. There is one interesting case which will appear in an applied setting quite often, however, which we will analyze in this section.

One fundamental problem of applied empirical work is to determine the set of explanatory variables to be included in the regression model. In the context of capital structure research, some variables are suggested by economic theory, but most empirical designs are driven by the findings of significant variables in previous studies. In the partial adjustment model, the included exogenous regressors determine the target leverage (see Equation (1)). It is well known that an “omitted variables bias” can seriously affect essentially all parametric estimators. A cautious researcher might thus include more explanatory variables into his design than predicted by theory or previous evidence, just to avoid omitting a relevant determinant of the data-generating process (he then sacrifices some of the estimator’s efficiency).

Table IV gives the results for one such case of over-specification in the setting of our dynamic panel fractional Monte Carlo design. Here it is assumed that the researcher includes a second explanatory variable (or capital structure determinant) in the regression specification, which under the true data-generating process is irrelevant. In the case of the DPF estimator this might have serious consequences because the model’s mis-specification
changes the presumed fixed effects distribution for estimation directly, since it is a function of the exogenous regressors and the initial condition.

**Table IV: Average Bias of Estimators in Mis-Specified Setup**

This table shows the deviation of the average estimate for each estimator relative to the true value of \( \lambda \) in the small sample simulation biases in %.

<table>
<thead>
<tr>
<th>True ( \lambda )</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>DPF</td>
<td>1.21</td>
<td>0.87</td>
<td>0.69</td>
<td>0.54</td>
</tr>
<tr>
<td>OLS</td>
<td>-21.89</td>
<td>-49.34</td>
<td>-51.39</td>
<td>-46.95</td>
</tr>
<tr>
<td>Fama/MacBeth</td>
<td>-88.08</td>
<td>-73.22</td>
<td>-62.72</td>
<td>-52.90</td>
</tr>
<tr>
<td>Arellano/Bond</td>
<td>171.50</td>
<td>57.76</td>
<td>31.03</td>
<td>21.78</td>
</tr>
<tr>
<td>Fixed effects</td>
<td>192.70</td>
<td>74.96</td>
<td>44.18</td>
<td>32.06</td>
</tr>
</tbody>
</table>

In the simulations underlying Table IV, we assume the regressions include a second exogenous variable with positive mean and correlation with the other exogenous variable \( z_{it} \), which is a component of the true data-generating process. For simplicity, we have modeled this superfluous variable as being the squared value of \( z_{it} \). As the table shows, the DPF estimator still yields nearly unbiased estimation results for the speed of adjustment compared to the results in Table III, indicating the robustness of the DPF estimator in this situation as well.

**B. Resampled Leverage Process**

Iliev and Welch (2010) suggest that one study the properties of estimators of the speed of adjustment by relying on the (unknown) true leverage distribution over time through resampling the observed data. The basic idea is to separate changes in the market value of equity from changes in leverage due to capital measures initiated by a firm’s management. This is implemented by (repeatedly) sampling the observed stock returns of a company and generating leverage paths over time by adding resampled capital changes (debt or equity net
issues) from randomly drawn (i.e. independent) firms in the sample. Note that this is a non-parametric alternative to the parametric data-generating process used in our simulation analysis of Section II. \(^\text{10}\)

In the partial adjustment model discussed in Iliev and Welch (2010),

\[
MDR_{i,t+1} = \lambda MDR^*_i + (1 - \lambda) MDR_{it},
\]

(4)

the \(MDR_{i,t+1}\) process is generated\(^\text{11}\) based on resampled observed debt changes \((\nu_{i,t,t+1}^{\text{rand}})\) and resampled observed equity changes \((\eta_{i,t,t+1}^{\text{rand}})\). The gross equity changes \((1 + \eta_{i,t,t+1}^{\text{rand}}) = (1 + r_{i,t,t+1})(1 + \kappa_{i,t,t+1}^{\text{rand}})\) consist of resampled net equity issuing changes \((\kappa_{i,t,t+1}^{\text{rand}})\) and the non-resampled originally observed stock returns without dividends \((r_{i,t,t+1})\). The resulting process for the debt ratio is

\[
MDR_{i,t+1} = \frac{MDR_{it}}{MDR_{it} + \frac{(1 + r_{i,t,t+1})(1 + \kappa_{i,t,t+1}^{\text{rand}})}{1 + \nu_{i,t,t+1}^{\text{rand}}}(1 - MDR_{it})}.
\]

(5)

This so-called “NULL”-process is the benchmark case to examine whether a given estimator is able to detect non-adjustment, if there is no adjustment under the data-generating process (i.e. the true \(\lambda\) equals zero). Note that this is the most important case for capital structure research, since then the trade-off theory would be economically irrelevant.

\(^\text{10}\)Iliev and Welch (2010) do not propose a new estimator as we do. They try to reconcile speed of adjustment estimates of existing estimators in their simulation analysis.

\(^\text{11}\)Due to the non-linear nature of debt ratios, certain assumptions have to be made for the special, but often observed, cases that debt changes are not defined (zero debt in \(t+1\) divided by zero debt in \(t\)) or are infinite (non-zero debt in \(t+1\) divided by zero debt in \(t\)). We deviate from the procedure through which Iliev and Welch (2010) account for these cases. We find that a Kolmogorov–Smirnov test cannot reject the null hypothesis that the original and resampled debt and equity changes using our procedure are drawn from the same distribution, whereas using the Iliev and Welch (2010) procedure does reject the null. The details of our mechanism are given in Appendix B.
The resampling procedure suggested by Iliev and Welch (2010) can also be used to study estimators’ characteristics if there is partial adjustment in the true data-generating process. This requires a target capital structure to be specified and implemented into the resampling procedure. Iliev and Welch (2010) use a target which is “one shock away” from the first observed debt ratio with fully resampled debt and equity changes, i.e. including resampled stock returns,

\[
MDR_{i,t+1}^* = \frac{MDR_{i0}}{MDR_{i0} + \frac{(1 + r_{t-1,t} + r_{t-1,t})}{1 + \nu_{t-1,t}}}, \tag{6}
\]

Figure 5 shows our simulation results of the partial adjustment model described in Equations (4), (5), and (6) using debt and equity changes from the Compustat sample described in Section IV.A. Again, we analyze the set of estimators as in our simulations of Section II, but we also include the so-called W estimator (Iliev and Welch (2010), Welch (2004)).

The left graph of Figure 5 shows simulation results where the true target leverage is included as an exogenous regressor in the regression specifications (“perfect knowledge of the target”). This setup is comparable to the one in our simulations in Section II. All applied methods overestimate the true speed of adjustment in these non-parametric simulations, though to a very different extent. Pooled OLS and Fama/MacBeth estimators perform worst. The also highly biased Arellano/Bond and fixed effects estimators reduce their biases

\[MDR_{i,t+1} = c + \lambda MDR_{it} + (1 - \lambda)IDR_{i,t+1} + \gamma MDR_{i,t+1} + u_{i,t+1},\]

where \(IDR_{i,t+1} = D_{it}/(D_{it} + (1 + r_{t+1})E_{it})\) is the implied debt ratio that results if the firm issues neither debt nor equity, that is, a stock return only induced debt ratio. The speed of adjustment in this specification is one minus the coefficient estimate on the \(IDR\) coefficient, because Iliev and Welch (2010) argue that it measures the degree to which firms readjust towards a target or undo stock return effects, respectively, whereas the coefficient on the lagged debt ratio might control for firm-specific targets.

\[\text{25}\]

\[\text{12}\] The regression specification of the W estimator that is only applicable in this specific capital structure context is
Figure 5: Simulation Results with Resampled Leverage Data

The figure shows the average estimates of the speed of adjustment $\lambda$, which corresponds to one minus the coefficient on the lagged dependent variable, i.e. $1 - \lambda$. The debt ratio data are generated according to the partial adjustment model described in Equations (4), (5), and (6) using debt and equity changes from our Compustat sample described in Section IV.A.

The horizontal axis shows the true parameter choice in the simulation and the vertical axis the corresponding estimated coefficient. Results are reported for different estimators: pooled OLS (“OLS”), Fama/MacBeth regressions (“FamaMac”), fixed effects (“FixEff”), Arellano/Bond (“AreBo”), the W estimator (“Welch”) and the DPF estimator suggested in Section II.A (“DPF”). The number of Monte Carlo runs is $S = 500$, where the number of firms is 16.170 and the number of firm years is 166.016 for each run. The first observed debt ratio for each firm in the sample is always the initial condition in the data-generating process for each simulation run.

The left graph shows simulation results where the true target leverage is included as an independent variable in the regression specifications (“perfect knowledge of the target”). The right graph shows simulation results where the true target leverage is omitted in the regression specifications (“unknown target”).

![Graph showing simulation results with resampled leverage data.](image-url)
for higher speeds of adjustment. The smallest biases show the W estimator and the DPF estimator,\(^{13}\) which also perform better for higher adjustment speeds. Compared to our simulation results based on the DPF estimator in Figure 2, the Arellano/Bond and fixed effects estimates are very similar; only the pooled OLS and Fama/MacBeth exhibit a different bias pattern, though comparable in magnitude. For most of the estimators our results are also similar to the ones found by Iliev and Welch (2010).

The right graph of Figure 5 shows simulation results where the true target leverage is omitted in the regression specifications, i.e. an “omitted variables bias” in terms of the exogenous regressors. Still, due to the inclusion of the lagged debt ratio as regressor, such a setting contains information on the target. The pooled OLS and Fama/MacBeth estimators are severely biased and now highly underestimate for most of the true speeds of adjustment, with increasing bias for higher speeds of adjustment. The fixed effects estimator clearly overestimates adjustment speed. The DPF estimator performs best, especially in the most relevant adjustment speed region \(\lambda = 0.0\) to \(\lambda = 0.4\). Except for the case of no readjustment towards a target \((\lambda = 0)\), the Arellano/Bond estimates perform second best in the relevant region. The W estimator nearly monotonically overestimates the true adjustment speed if target leverage is unknown.\(^{14}\)

\(^{13}\)The DPF regression specification in this context is given by

\[
MDR_{i,t+1} = \left[ \lambda MDR_{i,t+1}^* \right] + (1 - \lambda) MDR_{it} + c_i + u_{i,t+1},
\]

where unobserved heterogeneity is controlled for with

\[
c_i = a_0 + a_1 MDR_{i0} + \left[ \frac{1}{T_i} \sum_{t_i=1}^{T_i} MDR_{i,t_i}^* a_2 \right] + a_i.
\]

\(^{14}\)In the Iliev and Welch (2010) study, the omitted target only changes the pooled OLS estimates, leaving all other estimation results almost unchanged.
Unlike Iliev and Welch (2010), we find that (perfect) knowledge of the target does matter. Although much of the variation in debt ratios can be explained by the lagged dependent variable, not knowing the target—i.e. an omitted variables bias—has severe consequences in our adjustment speed estimation results as should have been expected.

We have conducted further simulations in which the target is not omitted but measured with error or in which we included an irrelevant target. We find that both tests reveal very similar effects on all estimators compared to the omission of the target. Given the results in this section and also the results from our mis-specification tests in Section III.A for the DPF simulations, we draw the same conclusion as Iliev and Welch (2010) that knowledge of the target does not matter (too much). However, we come to this conclusion for a different reason. As we have shown, perfect knowledge of the target clearly improves adjustment speed estimation (DPF estimator, \(W\) estimator), but in real data applications we will never be able to measure target leverage without error. As a conclusion, controlling for the initial condition problem, unobserved heterogeneity, and the fractional nature of debt ratios like our DPF estimator does becomes even more important to get least-biased adjustment speed estimates if target leverage cannot be measured without error.

Overall, the results of the robustness tests demonstrate that the DPF estimator performs well in the capital structure context. It is unbiased even with mis-specification of the underlying distributional assumption regarding the fixed effects. It shows the smallest bias of all examined estimators with resampled observed data, which reflect the true distribution of firm leverage under different scenarios of the true speed of adjustment and varying degrees of information available to control for the target debt ratio in the regressions.
IV. Empirical Analysis: Adjustment towards Target Leverage

A. Data

In this section, we apply the DPF estimator to real data on corporate finance, basically replicating the Flannery and Rangan (2006) analysis of adjustment behavior towards target leverage. The regression model is the partial adjustment specification in (3),

\[ MDR_{i,t+1} = \lambda \gamma X_{it} + (1 - \lambda) MDR_{it} + c_i + \epsilon_{i,t+1}, \]

as introduced in Subsection I.A. The vector of firm characteristics \( X_{it} \) determining target leverage \( MDR^{*}_{i,t+1} = \gamma X_{it} \) is chosen as in Flannery and Rangan (2006) and consists of

- **EBIT_TA**: (Profitability) – Earnings before interest and taxes divided by total assets.
- **MB**: (Growth Opportunities) – Market-to-book ratio of firm assets.
- **DEP_TA**: – Depreciation expense divided by total assets.
- **LnTA**: (Size) – Natural logarithm of total assets.
- **FA_TA**: (Asset Tangibility) – Fixed assets divided by total assets.
- **R&D_DUM**: Dummy variable equaling one for missing R&D expenses.
- **R&D_TA**: (R&D expenses) – R&D expense divided by total assets.
- **Ind_Median**: – Median market debt ratio of firm i’s Fama and French (1997) industry classification (plus the omitted SIC 3690) at time \( t \).
- **Rated**: (Rating) – Dummy variable equaling one for firms with public debt rating.
Flannery and Rangan (2006) use Compustat data for the period 1965 to 2001 in their analysis. To enable a comparison of results, we estimate the partial adjustment model for the full period from 1965 to 2008 and the sub-period from 1965 to 2001.

The sample consists of all industrial Compustat firms with complete data for two or more consecutive years during the period 1965 to 2008. Firms from the financial industry (SIC 6000–6999) and regulated utilities (SIC 4900–4999) are excluded. No firm size restrictions are imposed. The sample comprises 16,170 firms with 166,016 firm-years. All variables, except the market debt ratio $\text{MDR}_{it}$, are winsorized at the 1st and 99th percentile. For regression variables not being ratios, nominal magnitudes are expressed in 1983 dollars using the consumer price index as a deflator. Table V contains summary statistics on the sample for the years 1965 to 2008 and details of the construction of all employed variables. In the sample, about 10% of all (unwinsorized) market debt ratio observations are at zero, the lower limit of the potential market debt ratio range. Only 0.02% of all market debt ratios are at the upper limit of one. The 99% quantile of market debt ratios is roughly at an MDR of 92%.

B. Results

Table VI contains the speed of adjustment estimates for the partial adjustment model (3) over the period 1965 to 2008. Flannery and Rangan (2006) advocate the instrumental variables (IV) fixed effects estimator. This estimator uses book debt ratios ($\text{BDR}$) as instruments for lagged market debt ratios. The corresponding estimation result is reported in column 8 of Table VI. The coefficient on the lagged dependent variable, which is equivalent to one minus the speed of adjustment, shows at $100\% - 65.1\% = 34.9\%$ a relatively high speed of adjustment. A similarly high speed of adjustment is estimated by the fixed effects
Table V: Summary Statistics

The sample consists of all industrial Compustat firms with complete data for two or more consecutive years during 1965 to 2008. The number of firms is 16,170 and the number of firm years is 166,016. Market debt ratio \( (\text{MDR}_{it}) \) is not winsorized. All other variables are winsorized at the 1\(^{st}\) and 99\(^{th}\) percentile. The table also contains the absolute (and relative) number of market debt ratio observations that are both equal to zero and equal to one. Moreover, the 99\(^{th}\) quantile of the market debt ratio distribution is provided.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market debt ratio ( (\text{MDR}_{it}) )</td>
<td>0.2690</td>
<td>0.2059</td>
<td>0.2495</td>
<td>0.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>Book debt ratio ( (\text{BDR}) )</td>
<td>0.2460</td>
<td>0.2239</td>
<td>0.1980</td>
<td>0.0000</td>
<td>0.8161</td>
</tr>
<tr>
<td>Profitability ( (\text{EBIT_TA}) )</td>
<td>0.0059</td>
<td>0.0821</td>
<td>0.3031</td>
<td>-1.7758</td>
<td>0.3754</td>
</tr>
<tr>
<td>Market-to-book ( (\text{MB}) )</td>
<td>1.6808</td>
<td>1.0517</td>
<td>1.9760</td>
<td>0.2741</td>
<td>13.7143</td>
</tr>
<tr>
<td>Depreciation/Taxes ( (\text{DEP_TA}) )</td>
<td>0.0469</td>
<td>0.0384</td>
<td>0.0367</td>
<td>0.0006</td>
<td>0.2289</td>
</tr>
<tr>
<td>Size ( (\text{Ln_TA}) )</td>
<td>23.0484</td>
<td>22.8197</td>
<td>2.3497</td>
<td>18.4414</td>
<td>29.1547</td>
</tr>
<tr>
<td>Asset Tangibility ( (\text{FA_TA}) )</td>
<td>0.3101</td>
<td>0.2604</td>
<td>0.2263</td>
<td>0.0019</td>
<td>0.9028</td>
</tr>
<tr>
<td>No R&amp;D ( (\text{R&amp;D_DUM}) )</td>
<td>0.4607</td>
<td>0.0000</td>
<td>0.4884</td>
<td>0.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>R&amp;D expenses ( (\text{R&amp;D_TA}) )</td>
<td>0.0397</td>
<td>0.0000</td>
<td>0.1004</td>
<td>0.0000</td>
<td>0.7978</td>
</tr>
<tr>
<td>Industry Median MDR ( (\text{Ind_Median}) )</td>
<td>0.2237</td>
<td>0.2210</td>
<td>0.1349</td>
<td>0.0140</td>
<td>0.5815</td>
</tr>
<tr>
<td>Rating ( (\text{Rated}) )</td>
<td>0.1269</td>
<td>0.0000</td>
<td>0.3329</td>
<td>0.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th># Obs. Market debt ratio = 0</th>
<th># Obs. Market debt ratio = 1</th>
<th>99%-Quantile Market debt ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>17,444 (10.51%)</td>
<td>38 (0.22%)</td>
<td>0.9189</td>
</tr>
</tbody>
</table>

Market debt ratio \( (\text{MDR}_{it}) \): Book value of debt divided by the market value of assets (book value of debt plus market value of equity). \( \left( \frac{\text{Long-Term Debt}[9] + \text{Debt in Current Liabilities}[34]}{\text{Long-Term Debt}[9] + \text{Debt in Current Liabilities}[34] + \text{Price Fiscal Year Close}[199] * \text{Common Shares Outstanding}[25]} \right) \)

Book debt ratio \( (\text{BDR}) \): Book debt divided by total assets. \( \left( \frac{\text{Long-Term Debt}[9] + \text{Debt in Current Liabilities}[34]}{\text{Total Assets}[6]} \right) \)

Profitability \( (\text{EBIT\_TA}) \): Earnings before interest and taxes divided by total assets. \( \left( \frac{\text{Income Before Extraordinary Items}[18] + \text{Interest Expense}[15] + \text{Income Taxes}[16]}{\text{Total Assets}[6]} \right) \)

Market-to-book \( (\text{MB}) \): Market-to-book ratio of firm assets. \( \left( \frac{\text{Long-Term Debt}[9] + \text{Debt in Current Liabilities}[34] + \text{Preferred Stock}[10]}{\text{Total Assets}[6]} \right) \)

Depreciation/Taxes \( (\text{DEP\_TA}) \): Depreciation expense divided by total assets. \( \left( \frac{\text{Depreciation and Amortization}[14]}{\text{Total Assets}[6]} \right) \)

Size \( (\text{Ln\_TA}) \): Natural logarithm of total assets. \( \left( \text{ln} \left( \frac{\text{Total Assets}[6]}{1000000} \right) \right) \) measured in 1983 dollars, deflated by the consumer price index.

Asset Tangibility \( (\text{FA\_TA}) \): Fixed assets divided by total assets. \( \left( \frac{\text{Property, Plant, and Equipment}[8]}{\text{Total Assets}[6]} \right) \)

No R&D \( (\text{R\&D\_DUM}) \): Dummy variable equaling one for missing R&D expenses.

R&D expenses \( (\text{R\&D\_TA}) \): R&D expense divided by total assets. \( \left( \frac{\text{Research and Development Expense}[46]}{\text{Total Assets}[6]} \right) \)

Industry Median MDR \( (\text{Ind\_Median}) \): Median market debt ratio of firm \( i \)'s Fama and French (1997) industry classification (plus the omitted SIC 3690) at time \( t \).

Rating \( (\text{Rated}) \): (Debt market access) Dummy variable equaling one for firms with public debt rating. \( \left( \frac{\text{S&P LT Domestic Issuer Credit Rating}[280]}{\text{Total Assets}[6]} \right) \)

Numbers in brackets denote Compustat items.
estimator (column 6) at about 39%. A slower adjustment speed is estimated by pooled OLS, Fama/Macbeth, and the Arellano/Bond estimator. Recall that these estimates will be biased, given our simulation results in Section II.D.

More than 15,000 firm years (about 10% of all observations) have a market debt ratio of zero and there are a few observations with market debt ratios equal to one. The market debt ratio clearly is fractional and bounded between zero and one. The DPF estimator, taking this fractional nature of the market debt ratio into account, yields a speed of adjustment estimate of 26% (column 2). This result is in the middle of the range of adjustment speeds of previous studies. Column 3 in Table VI shows the speed of adjustment estimate of the DPF estimator, where the upper censoring limit is set to the 99% quantile of the market debt ratio rather than 1. At this upper censoring limit, 1,538 observations (about 1% of all observations) are censored, compared to 31 cases with censoring at one. The adjustment speed only changes marginally compared to the base DPF estimator. Hence, the DPF estimates are stable even for few observations at one of the censoring limits.

In our Monte Carlo simulations, we calibrated the simulated MDR distribution to resemble the corresponding real data distribution. Assuming a true speed of adjustment of 26%, as estimated for the real data by the DPF estimator, we can assess the potential bias of the other estimators for the real data by comparing their estimate with the corresponding simulation results.

For example, the Fama/MacBeth estimate is 14.3% ($1 - 0.857$) and thus lower than 26%. Similar to the simulations (see Figure 3), the speed of adjustment seems to be underestimated. The same holds for pooled OLS and the fixed effects estimator, though as a positive bias in the latter case. However, the estimate of the Arrelano/Bond estimator is lower than 26% in the real data (at about 19%), although the simulations would have predicted a pos-
Table VI: Comparison of Speed of Adjustment Estimates of Different Estimation Methods for Compu-stat Data

Regression results for the partial adjustment model of Flannery and Rangan (2006):

\[ MDR_{i,t+1} = (\lambda \gamma) X_{it} + (1 - \lambda) MDR_{it} + c_i + \epsilon_{i,t+1}, \]

where \( \lambda \) is the adjustment speed coefficient on the lagged market debt ratio (MDR), \( c_i \) a time-invariant unobserved variable (firm fixed effect), and \( \epsilon_{i,t+1} \) an error term. The (lagged) variables determining a firm’s long-run target leverage are described in Section IV and Table V. T-statistics are reported in parentheses and 99% \( -Q \). denotes the 99% quantile. ***,**,* at coefficient estimates denote significantly different from zero at the 1%, 5%, and 10% level, respectively.

<table>
<thead>
<tr>
<th></th>
<th>DPF [0,1]</th>
<th>DPF [0,99% – ( Q )]</th>
<th>Pooled OLS</th>
<th>Fama/MacBeth Fixed Effects</th>
<th>Arellano/Bond Fixed Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagged debt ratio (MDR_{it})</td>
<td>0.740***</td>
<td>0.744***</td>
<td>0.850***</td>
<td>0.857***</td>
<td>0.613***</td>
</tr>
<tr>
<td></td>
<td>(263.91)</td>
<td>(263.54)</td>
<td>(429.08)</td>
<td>(73.50)</td>
<td>(257.98)</td>
</tr>
<tr>
<td>Implied speed of adjustment ( \lambda )</td>
<td>26.0%</td>
<td>25.6%</td>
<td>15.0%</td>
<td>14.3%</td>
<td>38.7%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm years (N)</td>
<td>149,846</td>
<td>149,846</td>
<td>149,846</td>
<td>149,846</td>
<td>133,677</td>
</tr>
<tr>
<td>Upper censored value</td>
<td>149,846</td>
<td>149,846</td>
<td>149,846</td>
<td>149,846</td>
<td>133,677</td>
</tr>
<tr>
<td># Censored MDR obs. at 0</td>
<td>15,084</td>
<td>15,084</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># Censored MDR obs. at upper censored value</td>
<td>31</td>
<td>1,583</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs. period</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1965-2008</td>
</tr>
</tbody>
</table>
itive bias. As also suggested by Flannery and Rangan (2006), this effect might be due to a weak instruments problem of the Arellano/Bond estimator in the real data case. Note that the DPF estimate is roughly in the middle between the other estimates as in our simulations. This is another indication that the design of our Monte Carlo simulations generated data comparable to real MDR distributions.

Table VII shows the results for speed of adjustment estimates and the full set of firm characteristics determining target leverage for the period 1965 to 2001, the period originally used by Flannery and Rangan (2006). Column 2 provides our Flannery and Rangan (2006) replication, which again uses the instrumental variables (IV) fixed effects estimator. Our estimates differ just slightly from the original results. With regard to the DPF estimates (column 3), note that despite the huge difference for the speed of adjustment estimate, the coefficients and significance levels of the other explanatory variables barely change (only the rating dummy turns out to be insignificant). Finally, Table VII shows that the DPF estimates in general barely differ between the estimation periods (column 3 versus 4).

V. Conclusion

In this study we use Monte Carlo simulations to demonstrate that estimators commonly used in the literature to estimate the speed of adjustment of firms' capital structure are severely biased since they do not account for the fact that debt ratios are bounded between 0% and 100%. OLS and Fama/MacBeth regression estimates underestimate the true speed of adjustment by up to 50% and 90%, while fixed effects or the first difference GMM estimator by Arellano/Bond overestimate by up to 90%.

We suggest a new estimator, which is consistent in the context of unbalanced dynamic panel data with a fractional dependent variable (DPF estimator), and demonstrate its suit-
Table VII: Comparison of Regression Results for the DPF Estimator and the Flannery and Rangan (2006) Replication for Compustat Data

Regression results for the partial adjustment model of Flannery and Rangan (2006):

\[ MDR_{i,t+1} = (\lambda Y) X_{it} + (1 - \lambda) MDR_{it} + c_i + \epsilon_{i,t+1} \]

where \( \lambda \) is the adjustment speed coefficient on the lagged market debt ratio (MDR), \( c_i \) a time-invariant unobserved variable (firm fixed effect), and \( \epsilon_{i,t+1} \) an error term. The (lagged) variables determining a firm’s long-run target leverage are described in Section IV and Table V. T-statistics are reported in parentheses. ***, **, * at coefficient estimates denote significantly different from zero at the 1%, 5%, and 10% level, respectively.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Flannery and Rangan (2006) replication</th>
<th>DPF [0, 1]</th>
<th>DPF [0, 1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market debt ratio (MDR_{it})</td>
<td>0.635***</td>
<td>0.743***</td>
<td>0.740***</td>
</tr>
<tr>
<td></td>
<td>(170.25)</td>
<td>(228.00)</td>
<td>(263.91)</td>
</tr>
<tr>
<td>Profitability (EBIT_TA)</td>
<td>-0.038***</td>
<td>-0.009***</td>
<td>-0.016***</td>
</tr>
<tr>
<td></td>
<td>(-12.73)</td>
<td>(-2.81)</td>
<td>(-6.48)</td>
</tr>
<tr>
<td>Market-to-book (MB)</td>
<td>-0.000</td>
<td>0.004***</td>
<td>0.002***</td>
</tr>
<tr>
<td></td>
<td>(-0.21)</td>
<td>(9.78)</td>
<td>(7.60)</td>
</tr>
<tr>
<td>Depreciation/taxes (DEP_TA)</td>
<td>-0.219***</td>
<td>-0.292***</td>
<td>-0.231***</td>
</tr>
<tr>
<td></td>
<td>(-10.89)</td>
<td>(-13.73)</td>
<td>(-12.67)</td>
</tr>
<tr>
<td>Size (LnTA)</td>
<td>0.026***</td>
<td>0.014***</td>
<td>0.015***</td>
</tr>
<tr>
<td></td>
<td>(35.46)</td>
<td>(22.31)</td>
<td>(27.30)</td>
</tr>
<tr>
<td>Asset tangibility (FA_TA)</td>
<td>0.060***</td>
<td>0.060***</td>
<td>0.052***</td>
</tr>
<tr>
<td></td>
<td>(13.45)</td>
<td>(12.69)</td>
<td>(12.30)</td>
</tr>
<tr>
<td>No R&amp;D (R&amp;D_DUM)</td>
<td>-0.002</td>
<td>-0.001</td>
<td>-0.000</td>
</tr>
<tr>
<td></td>
<td>(-1.12)</td>
<td>(-0.51)</td>
<td>(-0.33)</td>
</tr>
<tr>
<td>R&amp;D expenses (R&amp;D_TA)</td>
<td>-0.037***</td>
<td>-0.016</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>(-3.48)</td>
<td>(-1.41)</td>
<td>(-0.87)</td>
</tr>
<tr>
<td>Industry median MDR (Ind_Median)</td>
<td>0.045***</td>
<td>0.025***</td>
<td>0.033***</td>
</tr>
<tr>
<td></td>
<td>(5.77)</td>
<td>(3.18)</td>
<td>(4.43)</td>
</tr>
<tr>
<td>Rating (Rated)</td>
<td>0.004**</td>
<td>-0.003</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(2.04)</td>
<td>(-1.64)</td>
<td>(1.21)</td>
</tr>
</tbody>
</table>

Firm years (N) 121,821 121,821 149,846
Upper censored value 1 1
# Censored MDR obs. at 0 10,045 15,084
# Censored MDR obs. at upper censored value 0 31
Obs. period 1965-2001 1965-2008
able statistical properties (unbiasedness and robustness against mis-specification) in the context of studies of capital structure. Moreover, we show that the DPF estimator performs best in the non-parametric resampling adjustment speed simulations setup of Iliev and Welch (2010).

We apply the DPF estimator to estimate the speed of adjustment for a typical sample of U.S. firms over the period 1965 to 2008, basically replicating the study by Flannery and Rangan (2006). The resulting estimate of 26% is in the middle of the range of adjustment speeds reported in previous studies. The associated half-life of leverage shocks of about 2.5 years does not rule out the economic relevance of the trade-off theory, but illustrates that alternative explanations for capital structure choices, like the pecking order theory or market-timing, are required to explain most of the observed variability in firm capital structures.

The DPF estimator we suggest is basically an extended doubly-censored Tobit-model and thereby rather easily implemented in standard econometric software packages. Since fractional variables occur quite often in corporate finance, the DPF estimator is applicable beyond capital structure research, as for example to the analysis of corporate payout policies.
Appendix

A. Stata Implementation of the DPF Estimator

The DPF estimator is easily implemented in Stata using the xttobit command. For example, if there are two exogenous regressors $z_{1it}$ and $z_{2it}$,

\[
\text{xttobit } y_{it} \ y_{i,t-1} \ z_{1it} \ z_{2it} \ y_{i0} \left[ \frac{1}{T_i} \sum_{t=0}^{T_i} z_{1it} \right] \left[ \frac{1}{T_i} \sum_{t=0}^{T_i} z_{2it} \right] , \quad \text{intm(gh)} \quad \text{ll(0)} \quad \text{ul(1)},
\]

gives the estimation results for the corresponding regressors, i.e. $(1 - \lambda)$ for $y_{i,t-1}$, $\gamma_1$ for $z_{1it}$, and so on. The estimated coefficient on the constant term is $\alpha_0$. The options specify using Gauss-Hermite quadrature (intm(gh)) and defining a lower limit at zero (ll(0)) and an upper limit at one (ul(1)).

B. Data-Generating Mechanism for the Resampled Leverage Process

As in Iliev and Welch (2010), every generated debt ratios sample in the simulations always has the same size (166,016 firm years). Each of the 16,170 firms in a generated sample has its initially observed debt ratio and the same number of periods as in the real data.
The $\text{MDR}_{i,t+1}$ process is generated based on resampled observed debt changes ($\nu_{i,t,t+1}^{\text{rand}}$) and resampled observed equity changes ($\eta_{i,t,t+1}^{\text{rand}}$),

\[
\text{MDR}_{i,t+1} = \frac{D_{i,t+1}}{D_{i,t+1} + E_{i,t+1}} = \frac{D_{it}(1 + \nu_{i,t,t+1}^{\text{rand}})}{D_{it}(1 + \nu_{i,t,t+1}^{\text{rand}}) + E_{it}(1 + \eta_{i,t,t+1}^{\text{rand}})} = \frac{MDR_{it}}{MDR_{it} + \frac{(1 + r_{i,t,t+1})(1 + \kappa_{i,t,t+1}^{\text{rand}})}{1 + \nu_{i,t,t+1}^{\text{rand}}}(1 - MDR_{it})}.
\]

Changes are resampled from the set of observed changes. Unlike Iliev and Welch (2010) we calculate real changes of debt and equity in order to not include changes caused by inflation. By construction, inflationary changes do not cancel out in the debt ratio, because randomly resampled debt and equity changes are not time-matched and usually will be taken from different firm year changes.

The gross equity changes $(1 + \eta_{i,t,t+1}^{\text{rand}}) = (1 + r_{i,t,t+1})(1 + \kappa_{i,t,t+1}^{\text{rand}})$ consist of resampled gross net equity issuing changes $(1 + \kappa_{i,t,t+1}^{\text{rand}})$, which is the change in common shares outstanding, and the non-resampled originally observed gross stock returns without dividends $(1 + r_{i,t,t+1})$, which is the change in the deflated price fiscal year closed. We do not use CRSP stock returns without dividends but calculate them from the Compustat data in the same way as it is done in CRSP, that is, we do not have to match databases and can keep all firm years in our sample. Iliev and Welch (2010) also checked this variant and report that their estimation results do not change. For the definitions of debt $D$, equity $E$, *Common Shares Outstanding*, *Price Fiscal Year Closed*, and the deflator, see Table V.

Resampling debt ratios based on debt and equity changes is intended to draw samples from the originally observed leverage distribution. Due to the non-linear nature of debt ratios, we have to make assumptions if debt changes in Equation (B1) are not defined (zero
debt in \( t + 1 \) divided by zero debt in \( t \)) or are infinite (non-zero debt in \( t + 1 \) divided by zero debt in \( t \)). We deviate from the procedure whereby Iliev and Welch (2010) account for these cases.

We have 3,397 observed infinite (Inf) changes and 11,763 observed not defined (NaN) changes. If a firm has lagged zero leverage we randomly draw an Inf change with 22.41% probability or a NaN change with 77.59% probability according to the observed debt changes involving zero leverage. If we draw a NaN change—that is, the firm stays at zero leverage—we set the current leverage to zero again. If we draw an Inf change—that is, the zero leverage firm takes on debt—we randomly draw from set of all debt ratios observed after an Inf change and set the current leverage to this debt ratio. For Inf changes, Iliev and Welch (2010) use a different procedure. They winsorize large debt and equity changes whereas we do not.

We find that a Kolmogorov–Smirnov test cannot reject the null hypothesis that the original and resampled debt and equity changes using our procedure are drawn from the same distribution, whereas using the Iliev and Welch (2010) procedure does reject the null.

For higher adjustment speeds from \( \lambda = 0.1 \) to \( \lambda = 0.9 \) we get lower means starting from about 26 to 20% and standard deviations starting from 29 to 24% for simulated debt ratios and approximately the same number of about 18,500 zeros and about 70 ones. These figures are consistent with the observed leverage distribution and we get similar results in our DPF simulations in Table I. Only for zero adjustment speed \( \lambda = 0.0 \) do we get a fairly high average mean (28 %) and standard deviation (34 %), and a much larger number (3,273) of ones.
References


Baltagi, Badi H., 2005, *Econometric Analysis of Panel Data* (John Wiley & Sons Ltd.).


